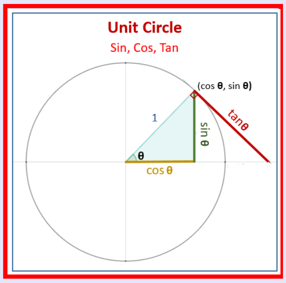
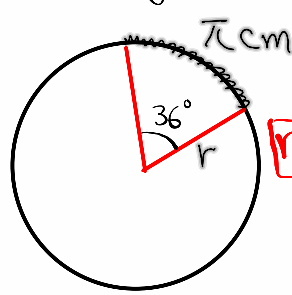
Math 241

Winter 2023

Lecture 7



If the Central angle of a Sector is 36°
with arc length of π cm. Drawing Required.



$\boxed{r = 5\text{cm}}$

1) find its radius.

$S = r\theta$

$\pi = r \cdot \frac{\pi}{5}$

$5\pi = \pi r$

$\frac{5\pi}{\pi} = r \rightarrow r = 5$

$180° = \pi$ Rad.

$1° = \frac{\pi}{180}$ Rad.

$36° = \frac{36\pi}{180}$ Rad

$36° = \frac{\pi}{5}$

2) find its area.

$A = \frac{1}{2} r^2 \theta$

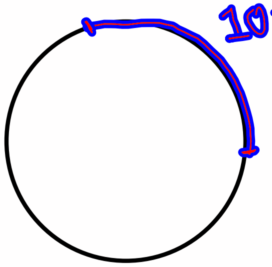
$= \frac{1}{2} \cdot 5^2 \cdot \frac{\pi}{5} = \frac{25\pi}{10}$

$= \boxed{2.5\pi \text{ cm}^2}$

Linear Velocity $v = \frac{s}{t}$ $\Rightarrow$ $\omega = \frac{v}{r}$

Angular Velocity $\omega = \frac{\theta}{t}$ $v = r\omega$

Find v if $s = 10\pi$ cm with $t = 2$ minutes.



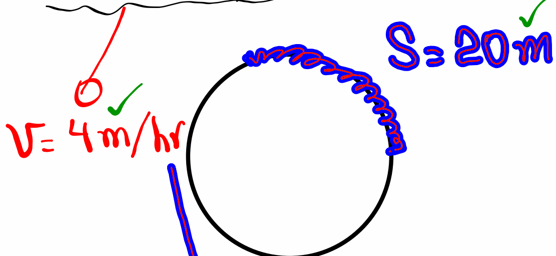
$v = \frac{s}{t} = \frac{10\pi}{2} = 5\pi$ cm/m.

What about per hr?

$\frac{5\pi \text{ cm}}{1 \text{ Minute}} \cdot \frac{60 \text{ minutes}}{1 \text{ hr}} =$

300π cm/hr

How long does it take for an object to travel on a circular path 20m with linear speed of 4 m/hr.?



$v = \frac{s}{t}$

$4 = \frac{20}{t}$

$4t = 20$

$t = \frac{20}{4}$

$\boxed{t = 5}$ hr

Find the angular velocity on a circular Path with central angle of 45° in 2mins?

Radians

$180° = \pi$

$1° = \frac{\pi}{180}$

$45° = \frac{45\pi}{180} = \frac{\pi}{4}$

$\omega = \frac{\theta}{t} = \frac{\frac{\pi}{4}}{2 \text{ mins}}$

$= \frac{\pi}{4} \div 2 = \frac{\pi}{4} \div \frac{2}{1}$

$= \frac{\pi}{4} \cdot \frac{1}{2} = \frac{\pi}{8}$ Rad./min.

$\omega = \frac{\pi}{8}$ Rad./min.

What about degrees per Second?

$180° = \pi$

$\frac{\cancel{\pi} \cancel{\text{Rad}}}{8 \cancel{\text{mins.}}} \cdot \frac{\overset{3}{\cancel{180}} \text{ deg.}}{\cancel{\pi} \cancel{\text{Rad}}} \cdot \frac{1 \cancel{\text{mins.}}}{\cancel{60} \text{ Seconds}} = \frac{3}{8}$ deg/Sec.

Find ω if $\theta = 45\pi$ and $t = 1.2$ hrs.

$\omega = \frac{\theta}{t} = \frac{45\pi}{1.2} = 37.5\pi$ Rad/hr

Given $\omega = 3\pi/2$ Rad/Sec, $r = 4m$, $t = 30$ Sec.

1) Find S

$\omega = \frac{\theta}{t}$ $\quad \frac{3\pi}{2} = \frac{\theta}{30}$

$2\theta = 30(3\pi)$

$\theta = \frac{30(3\pi)}{2}$

$\boxed{\theta = 45\pi}$ Rad.

$S = r\theta$

$= 4(45\pi) = \boxed{180\pi \ m}$

2) Find the area of that Sector.

$A = \frac{1}{2} r^2 \theta$

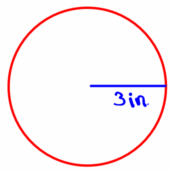
$= \frac{1}{2} \cdot 4^2 \cdot 45\pi = \boxed{360\pi \ m^2}$

A circular device has a radius of 3 inches,

It is turning at $\boxed{600 \text{ Revolutions per minute}}$

what is the linear speed of one point

on the edge in $\boxed{\text{Feet per minute}}$



$\omega = 600 \ \underbrace{\text{Revolutions}}/\text{min.}$

$= 600\,(2\pi)$ Rad./min.

$= 1200\pi$ Rad./min.

$v = \dfrac{s}{t}$ $\quad \omega = \dfrac{\theta}{t}$ $\Rightarrow$ $\omega = \dfrac{v}{r}$

$v = r\omega = 3\text{in.} \cdot 1200\pi/\text{min} = 3600\pi$

$\boxed{v = 3600\pi \text{ in/min.}}$

$\dfrac{\cancel{3600}^{3}\pi \ \cancel{\text{in.}}}{1 \ \text{Min.}} \cdot \dfrac{1 \ \text{ft}}{\cancel{12} \ \cancel{\text{in.}}} = 300\pi \ \text{ft/min.}$

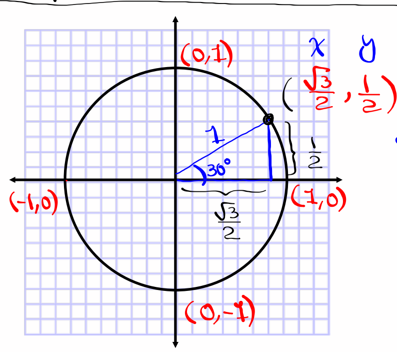
what about ft/sec?

$\dfrac{\cancel{300}^{5}\pi \ \text{ft}}{1 \ \cancel{\text{min}}} \cdot \dfrac{1 \ \cancel{\text{Min}}}{\cancel{60} \ \text{Sec.}}$

$\boxed{5\pi \ \text{ft/sec.}}$

Recall the chart below:

	$0°$	$30°$	$45°$	$60°$	$90°$
Sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	Und.



$\sin 0° = 0$ $\quad$ $\sin 90° = 1$

$\cos 0° = 1$ $\quad$ $\cos 90° = 0$

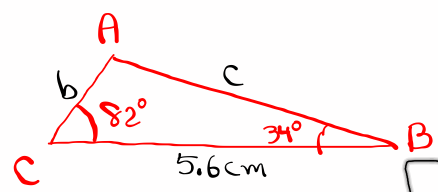
$\tan 0° = 0$ $\quad$ $\tan 90°$ undefined

$\sin 180° = 0$ $\quad$ $\sin 270° = -1$ $\quad$ $\sin 360° = 0$

$\cos 180° = -1$ $\quad$ $\cos 270° = 0$ $\quad$ $\cos 360° = 1$

$\tan 180° = 0$ $\quad$ $\tan 270° = \text{Und.}$ $\quad$ $\tan 360° = 0$

Solve triangle ABC if B = 34°, C = 82°, and a = 5.6 cm. Drawing required.



$A + B + C = 180°$

$A + 82 + 34 = 180$

$\boxed{A = 64°}$

Using Law of Sines

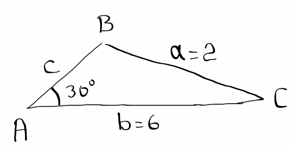
$$\boxed{\frac{\sin 64°}{5.6} = \frac{\sin 34°}{b}} = \frac{\sin 82°}{c}$$

$$\frac{\sin 64°}{5.6} = \frac{\sin 34°}{b} \Rightarrow b = \frac{5.6 \sin 34°}{\sin 64°} \qquad \boxed{b = 3.5\text{cm}}$$

$$\frac{\sin 64°}{5.6} = \frac{\sin 82°}{c} \Rightarrow c = \frac{5.6 \sin 82°}{\sin 64°} \qquad \boxed{c = 6.2\text{cm}}$$

Solve $\triangle ABC$ if $a = 2$in, $b = 6$in, and $\angle A = 30°$.

Drawing Required.



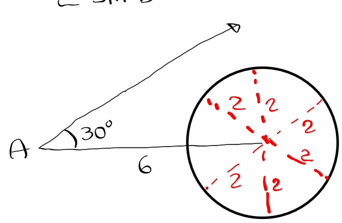
Using Law of Sines

$$\boxed{\frac{\sin 30°}{2} = \frac{\sin B}{6}} = \frac{\sin C}{c}$$

$2 \sin B = 6 \cdot \sin 30°$

$2 \sin B = 6 \cdot \frac{1}{2}$

$2 \sin B = 3$



$\sin B = \frac{3}{2} = 1.5$

$B = \sin^{-1}(1.5)$

No Solution

$-1 \leq \sin \alpha \leq 1$

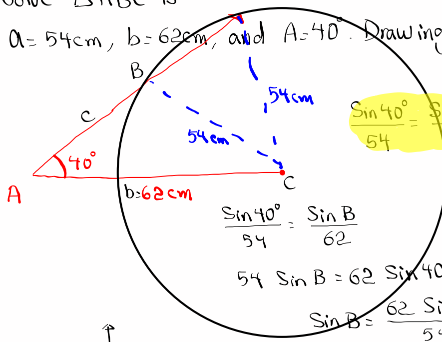
$-1 \leq \cos \alpha \leq 1$

Side a is not long enough to intersect the other side

No such triangle

Solve $\triangle ABC$ if

$a = 54cm$, $b = 62cm$, and $A = 40°$. Drawing required.



$\frac{\sin 40°}{54} = \frac{\sin B}{62} = \frac{\sin C}{c}$

$\frac{\sin 40°}{54} = \frac{\sin B}{62}$

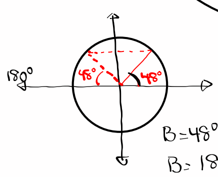
$54 \sin B = 62 \sin 40°$

$\sin B = \frac{62 \sin 40°}{54}$

$\sin B = .738$

$B = \sin^{-1}(.738)$

$B \approx 48°$



$B = 48°$

$B = 180° - 48° = 132°$

Case I: $B = 48°$

$A = 40°$

$A + B + C = 180°$

$40 + 48 + C = 180°$ $\boxed{C = 92°}$

Case II $B = 132°$

$A = 40$

$40 + 132 + C = 180°$

$\boxed{C = 8°}$

There are two Possibilities.

$\frac{\sin 40°}{54} = \frac{\sin 92°}{c}$

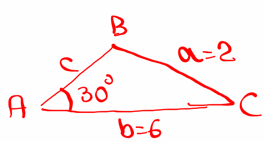
$c = \frac{54 \sin 92°}{\sin 40°}$ $c \approx 84cm$

$\frac{\sin 40°}{54} = \frac{\sin 8°}{c}$

$c = \frac{54 \sin 8°}{\sin 40°}$ $c \approx 12cm$

Use law of Cosines to Solve for $\triangle B$ in

$\triangle ABC$ if $a = 2$, $b = 6$, and $A = 30°$.



$b^2 = a^2 + c^2 - 2ac \cos B$

$a^2 = b^2 + c^2 - 2bc \cos A$

$2^2 = 6^2 + c^2 - 2(6) \cdot c \cdot \cos 30°$

$4 = 36 + c^2 - \cancel{12}^{6} c \cdot \frac{\sqrt{3}}{\cancel{2}}$

$c^2 - 6\sqrt{3}\, c + 36 - 4 = 0$

$c^2 \underbrace{- 6\sqrt{3}}\, c + 32 = 0$

$a = 1$ $b = -6\sqrt{3}$ $c = 32$

$b^2 - 4ac =$

$(-6\sqrt{3})^2 - 4(1)(32) =$

$36 \cdot 3 - 4 \cdot 32 =$

$108 - 128 = -20$

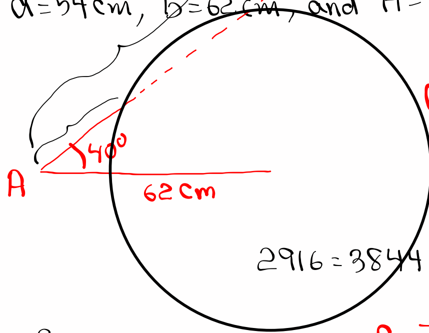
$c = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\sqrt{-20}$ undefined

No such triangle

Solve ΔABC if

$a = 54\text{cm}$, $b = 62\text{cm}$, and $A = 40°$. Use Law of Cosines



$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$54^2 = 62^2 + c^2 - 2(62)c \cdot \cos 40°$$

$$\vdots$$

$$2916 = 3844 + c^2 - 95c$$

$$c^2 - 95c + 928 = 0$$

$a = 1$ $\quad b = -95$ $\quad c = 928$

$$b^2 - 4ac = (-95)^2 - 4(1)(928)$$

$$= 5313$$

$$c = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-95) \pm \sqrt{5313}}{2}$$

$$= \frac{95 \pm 73}{2}$$

$$c = \frac{95 + 73}{2} \approx 84\text{cm}$$

$$c = \frac{95 - 73}{2} \approx 11\text{cm}$$

Given $a = 4m$, $b = 6m$, and $c = 8m$

Find one of the angles of ΔABC.

$$a^2 = b^2 + c^2 - 2bc\cos A \implies \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$b^2 = a^2 + c^2 - 2ac\cos B \implies \cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$c^2 = a^2 + b^2 - 2ab\cos C \implies \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos A = \frac{6^2 + 8^2 - 4^2}{2(6)(8)} = \frac{84}{96} = .875$$

$\cos A = .875$ $\qquad A = \cos^{-1}(.875)$

$$\boxed{A \approx 29°}$$

$$\cos B = \frac{4^2 + 8^2 - 6^2}{2(4)(8)} = \frac{44}{64} = .6875$$

$$B = \cos^{-1}(.6875)$$

$$\boxed{B \approx 47°}$$

$$\cos C = \frac{4^2 + 6^2 - 8^2}{2(4)(6)} = \frac{-12}{48} = -.25$$

$$C = \cos^{-1}(-.25) \approx \boxed{104°}$$

Let's verify that

$$A + B + C = 180°$$

$$29° + 47° + 104° = 180°$$